Math Development



Introduction: Mathematic attainment levels can vary significantly amongst students with Down syndrome; however most students have number skills approximately two years behind their reading levels. One reason is that mathematical concepts involve abstract reasoning and performance of simultaneous cognitive operations. For example, one number or concept must be maintained in memory while a computation is simultaneously performed. Most individuals with DS do not progress much beyond the concrete operational level of cognition, making abstract concepts (e.g. such as using 'x' and 'y' to represent numbers in algebra) difficult to master. Drills and memorization of facts may be controversial, but some experts argue that mastery of skills until they become automatic will "free up" space in working memory, which is the mental workspace used for calculations and problem-solving.

Furthermore, ordering, comparing and describing features of items are the foundation of many mathematical skills, which can be affected by language delays. For example, subtraction may be referred to as "subtract", "take away", "fewer" or "less than". Descriptive words may differ for the same concept, such as, "big, bigger, and biggest" also being referred to as "small, medium, and large".

Once number recognition and number order is established, 1:1 correspondence is typically taught. Once students can count objects in a line, practice counting objects laid in different patterns, which require students to keep track of counted items. You can use predictable patterns (e.g. dice, dominoes) and learn to 'spot count', so students can identify 'how many' without having to count each item. Most students can learn to spot count up to 4 - 6 items.

To build on understanding 1:1 correspondence, request a specific number of items from the student (e.g. "Give me 3 crackers."). The written numeral should be shown, so students don't forget what number amount was asked for. Having the number symbol in view for students will help them stop when their count matches the number requested.

Number lines (1-10, then 1-20) are highly effective and students can place tokens or colored paperclips on starting numbers and move the second clip up and down the number line.

Numbers 11, 12 and 13 are best taught separately from other 'teen' numbers and numbers greater than 10 are best presented in number grids where the numbers align to promote patterns and skip counting. Skip counting by 2s, 5s, and 10s are important and help with money, time and more efficient counting.

Addition/Subtraction: When adding two digits, teach students to start with the largest number and 'add on' the smaller number using a number line. Many students have a tendency to want to start at zero and count up to the largest of the two numbers rather than starting with the larger number. Students must have a good understanding of greater and lesser numbers. Addition should be taught first, then subtraction. For example, teach adding with sums up to 5, then subtraction up to 5. Next teach addition with sums to 10, then subtraction within 10 and so on.

Memorizing addition fact families (facts adding up to 10) can be very valuable, but also take some time. Once these are memorized however, it can save valuable time and free up some working memory for more complex operations. Speed card and dice games can be a fun way to practice adding to 10 or 12 quickly.

The Numicon system is also recommended for visual learners, which is based upon a ten-set and uses a 100-frame. Numicon uses colored tiles that provides additional visual discrimination along with easy-to-use manipulatives. Fine motor delays and challenges can make use of math manipulatives frustrating for some children and can become counterproductive. Therefore minimize fine motor demands during math work. For example, provide a scribe to write numbers or use number tiles or a keyboard.

Students will likely need the same amount of time (if not more) to learn subtraction. Firstly, students cannot see what is taken away in subtraction the same way they can when things are added. Secondly, students do not receive as much practice counting backward as they do forward.

Multiplication: Many students find it easier to memorize multiplication facts, so they can retrieve these automatically as difficulties can arise when students need to organize a problem before solving it. Knowing multiplication facts is essential for solving problems quickly. The 2s, 5s, and 10s are useful for many daily activities. When multiplication is understood, so that the student understands the computation procedure and the symbol used for multiplication (X), a calculator can be used to find the solution. Some students relate better to multiplication when they realize it is a faster method than addition to compute a sum.

Division: The strategies for division are similar to those used for multiplication. Many students learn best by remembering the multiplication/division fact family (e.g. $2 \times 3 = 6$, $3 \times 2 = 6$, 6 / 2 = 3 & 6 / 3 = 2). However, it is essential that students understand that the bigger number is divided into groups of smaller numbers. Often, the concept to dividing items (e.g. cookies) fairly among individuals (e.g. their friends) resonates with students putting division into a more meaningful context.

Grouping and Place Value: Multiple practice sessions with various items may be required to teach bundling 10 items (popsicle sticks, toothpicks, cards, pipe cleaners, clothespins, etc) to represent one item in the tens place. Children also need practice to see how tens fit into a 100 square. The 100 square should have numbers arranged from 1-10 across the top with 11-20 printed under corresponding place and so on. This way children can count neatly by tens by moving vertically down the columns.

Color or shape changes can be used to show hundreds, tens and ones. For example have a student exchange 10 white blocks for one red block and 10 red blocks for one blue block. Writing the headings for the columns (e.g. hundreds, tens, ones) above where numbers are to be placed also serves as a visual cue and reminder.

Whole Number Addition: The process of regrouping is a major step in moving forward with more complex addition and can be challenging for many students. While some students may learn this concept through practice, others may be better suited to use a calculator for numbers beyond 9. When using a calculator, the focus should be on entering the numbers correctly, especially when there are several, which can be difficult due to memory issues and tracking one's place in number entry. As with all calculator skills, the student should have a clear understanding of the computation being utilized (in this case, addition.

Whole Number subtraction: Use of a number line to with sums up to 20 is recommended, with use of a calculator when sums are greater than 20. When adding columns of numbers, use large graph papers to align numbers or use different colored pencils for numbers of different place value. It is important for students to remember that smaller numbers are subtracted from larger numbers; therefore being able to compare numbers is a prerequisite.

It is also important for students to know that ones are to be subtracted from ones, tens from tens and hundreds from hundreds. Returning to the use of items may be needed where students can unbind a ten-bundle in order to 'borrow' items so they can subtract from like units. Although a calculator may need to be used earlier in this learning process than with addition, it is still important that students understand the regrouping and borrowing process that this computation is based upon.

Word Problems: Picture or object representations of word problems can be very effective. Students can use stamps or draw their own items to be computed. Use of graphic organizers is a valuable teaching device that can be faded as students build competence. For example, have blocks already printed on the page where students can draw, stamp or place their items. Tally marks can also serve as a visual representation and can students practice counting by fives.

Also, using declarative sentences to fill in the answer ("He had _____ apples.") may be a more effective prompt than asking an open-ended question ("How many apples did he have?").

Recommended steps to completing a word problem:

 Read the Problem, 2) Reword the problem in the student's own words, 3) reword each question as a statement, 4) draw or act out the problem (if possible), 5) decide what operation will be used, 6) write the number sentence, 7) solve the problem using a calculator, if necessary, 8) check the answer.

Time: Understanding time concepts can be difficult as time is abstract you can't see what you are measuring. Furthermore, the measurement of time does not follow an orderly fashion by tens or twenties. For example, there are 60 minutes in a n hour, 24 hours in a day, 7 days in a week, 4 weeks in a month, etc. Use of daily, weekly, monthly and annual calendars can give visual support for understanding carrying lengths of time, especially when they are linked to student's own life events. Talking about (and marking on a schedule and calendar) past and future events can be valuable and motivating way to teach these concepts.

It is recommended to use both analogue and digital clocks when learning to tell time. Although a digital clock is easier to read, an analogue clock provides a visual sense of time. Clocks that have the minutes marked by fives placed outside the 1-11 can be helpful, to help them remember what the second-hand placement represents. The recommended sequence of learning to tell time consists of teaching the hour with only the hour hand, then with 30 minute increments, then 15 minute increments, then five minutes, and so on. Students should avoid learning expressions such as "a quarter after" or "half past" as these can be confusing.

Money: Playing 'buying' games with real or play money can be extremely helpful, especially when using realistic costs. Placing stickers or writing the value on one side of coins can also be helpful until coin amounts are memorized. Adding coins to arbitrary amounts (e.g. 83 cents), is less functional in everyday life. For some students it can also be a time consuming and frustrating exercise compared to rounding up to the next dollar and estimating one's change.

Prerequisites for functional money skills include knowing the name and value of common coins and bills; matching coins and bills to items that cost that amount; skip counting by 5s, 10s, and 25s up to 100; counting pennies to 25; reading prices that have both dollars and cents; rounding up money amounts to the next highest dollar; and using the next highest dollar strategy for prices that have both dollar and cents.

Secondary money skills include: recording money spent; keeping track of one's balance; planning weekly expenses; budgeting earning and expenses weekly; filling out checks or using a debit card; and understanding how credit works.

Measurement: Piaget's concept of Conservation is important when learning about measurement. Students need to understand that no matter how they arrange, move or cut an item, its total quantity is the same. For children who have not yet reached this developmental level, these concepts may be difficult. Measurement concepts are best learned over time and through a variety of hands-on experiences, including science and cooking activities.

Use consistent terms to help discriminate what type of measurement you are using. For example, Length (How long), Capacity (How much), Weight (How heavy), Temperature (How hot).

Students also have to recognize comparison terms, such as big, bigger, biggest or small, medium, large. The various vocabulary associated with length, for example, can include width, depth, height, tall, long, short, narrow. These often mean something different depending on context, which makes practice in various situations important. Measuring items relative to each student is helpful, such as the length of one's foot compared to classroom items or other student's feet.

Using hands-on cooking and science activities can be a highly effective way to teach concepts of capacity, weight and temperature. Use of standard weight or balance scales give students visual representation of these concepts.

Graphs: Graphing is often a more easily understood concept and activity due to its visual representation of concepts. Visually representing ideas throughout the day in different forms can be helpful. For example, after the class votes on the snack or book they want read, show the votes in tally marks and in graphical representation. It is suggested to color code or have picture supports as needed for graphs.

Simple Fractions: Understanding the visual representation of fractions (e.g. half of an apple) is much easier than understanding the written form of fractions (1/2), especially when fractions are written in a variety of ways (numerator to the left if the denominator, numerator above the denominator). Introducing fractions through concrete hands-on activities (e.g. cooking) can be very helpful. Learning the language of fractions; however can be challenging. For example, 1/4 = one fourth, 1/5 = one fifth; however $\frac{1}{2}$ does not equal one "twoth" nor does 1/3 equal one "threeth".

Geometry: Memorizing the equations and shape names in geometry may be easier than other mathematical processes. It may also be easier for some students to use common versus technical terms for geometrical objects, such as ball, box or tube instead of sphere, cube or cylinder. Similarly, having students become familiar with the vocabulary of geometry before equations are performed may be helpful, such as the terms: angle, line, parallel, perpendicular, circumference, diameter, and radius.

Decimal Fractions and Percents: Understanding the relationship between fractions, decimals and percents is an important concept for understanding money and shopping. It is important to teach about hundredths since this concept relates to both money and percent.

Visualizing a "wall" at the decimal point may be helpful, as well as verbally and visually differentiating between "hundreds" and "hundredths" or between "tens" and "tenths".

Have students practice naming mixed numbers by saying, "and" when they get to the decimal place. For example: 4.36 = 4 and thirty-six hundredths or 3.04 = 3 and 4 hundredths or 6.8 = 6 and 8 tenths.

It is also important for students to convert a two-place decimal into a percent and vice versa. Using matching or Bingo games to do this can be fun way to practice. Calculation of percent off while shopping is another valuable survival skill and use of a calculator is recommended for this multi-step process. Tip chart to calculate restaurant tips may also be helpful and efficient.

References:

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